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Extra dimensions, warped compactifications and cosmic acceleration

Ishwaree P. Neupane

Department of Physics and Astronomy, University of Canterbury, Private Bag 4800, Christchurch 8041, New Zealand, E-mail: ishwaree.neupane@canterbury.ac.nz, and Kavli Institute for Theoretical Physics China, CAS, Beijing 100190, China

We report on explicit cosmological solutions within the framework of an inflating de Sitter brane embedded in five- and ten-dimensional bulk spacetimes. In the specific example we study the brane tension is induced by the curvature related to the expansion of a physical $3+1$ spacetime rather than by a bulk cosmological term. In a generic situation with nonzero brane tension, the expansion of the universe accelerates eventually. We also show that inflationary cosmology is possible for a wide class of metrics without violating four- and higher-dimensional null energy condition.

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I. INTRODUCTION

The one major development that was not anticipated was the discovery that the expansion of the today's universe is accelerating [1], rather than slowing down. Since an epoch of cosmic acceleration plays an important role in modern cosmological models, it would be very interesting to know whether or not this effect can be understood or explained within the framework of fundamental theories, including superstring and supergravity models.

In recent years, several attempts have been made to find explicit cosmological solutions of ten- and eleven-dimensional supergravity models that allow accelerating universes using time-dependent scalar fields or metric moduli [2, 3, 4]. Time-dependent solutions in pure supergravity generally require some of the extra spaces to be negatively curved, if they are to allow a cosmic acceleration of the usual $3+1$ spacetime.

There are a couple of disadvantages of using explicit time-dependent scalar fields. First, in many examples studied in the literature, with maximally symmetric extra dimensions, we usually obtain only a transiently accelerating universe with time-dependent volume moduli, see e.g. [3, 5, 6]. Second, cosmological solutions with time-dependent scalar fields usually contain time-like singularities. This last feature (of a cosmological solution) is generally unacceptable because generic singularity of a time-dependent solution in pure supergravity may not have any quantum interpretation.

In [7] it was first realized that cosmological solutions without any time-like singularities can be obtained by introducing one or more geometric twists in the extra dimensions which generate in lower dimensions some nontrivial metric flux. Yet the corresponding solutions do not lead to a four-dimensional de Sitter (or quasi de Sitter) spacetime as is required to describe the inflationary epoch of the universe at its early stages and/or the present universe with a period of accelerating expansion. We therefore seek to an alternative scenario with warped extra dimensions.

In 1999, Randall and Sundrum in a theory referred to as RS1 [8] realized that a five-dimensional braneworld model with a brane can address the mass hierarchy in particle physics if there is a second brane some distance away from the first, which perhaps mimics the observed $3+1$ spacetime. An even more revolutionary idea was that gravity can be 'trapped' (on a brane) and extra dimensions may have infinite spatial extent [9]. For this simple and elegant proposal to work one needs a five-dimensional anti de Sitter space, i.e. a background geometry which is negatively curved, which suppresses the effect of warping at the brane's position or the 4D hypersurface, leading to a zero cosmological constant. Once the bulk cosmological term is assumed to be zero then the RS solution would be lost. It is therefore of natural importance (and our interest) to find nontrivial (cosmological) solutions that exist in flat spacetimes as well.

One simple thing that can happen when we view our observed universe as a cosmological brane embedded in a higher-dimensional spacetime is that the universe can accelerate because of an effective four-dimensional cosmological constant induced on the brane or due to the warping of additional spatial dimensions. In general, this phenomenon can (and should perhaps) occur when the tension on the brane(s) is positive.

Following [8, 9], we find interest in warped metrics that maintain the usual four-dimensional Poincaré symmetry, with general metric parametrization:

$$ds_D^2 = W(y)^2\, \hat{g}_{\mu\nu} dX^\mu dX^\nu + W(y)^\gamma g_{mn}(y)\, dy^m dy^n, \quad (1)$$

where X^μ are the usual spacetime coordinates ($\mu, \nu = 0, 1, 2, 3$), $W(y)$ is the warp factor as a function of one of the internal coordinates and γ is a constant. Nonfactorizable metrics as above can be phenomenologically motivated as in five-dimensional braneworld models as well as in ten- and eleven-dimensional supergravity models with reduced (super)symmetries [10, 11, 12]. They can also arise naturally in string theory compactification with flux [13, 14, 15, 16].

In this Letter by considering the metric (1), we present explicit cosmological solutions for which not only the warp factor is nontrivial but also the physical $3+1$ spacetime undergoes an inflationary de Sitter expansion, especially, when the brane tension is nonzero. An intriguing

feature of such new solutions is that the scale factor of the universe becomes a constant only in the limit where the warp factor $W(y)$ also becomes a constant. In a sense, the warp factor cannot be a constant except in the region where the scale factor of the universe is also constant, leading to a Minkowski spacetime.

For generality, we take the full spacetime dimensions to be D, which we split as $D \equiv 4+m \equiv 4+1+q$. The internal m-dimensional manifold is assumed to be an Einstein space

$$ds^2_{D-4} = g_{mn}(y)\, dy^m dy^n$$

having positive, negative or zero Ricci scalar curvature ($R^{(m)} > 0$, $R^{(m)} < 0$ or $R^{(m)} = 0$). We should note that the choices made by Gibbons [10], Maldacena and Nunez [12] and Giddings et al. [15], with respect to warped compactifications, are all different. These are, respectively, $\gamma = 0$, $\gamma = 2$ and $\gamma = -2$. This difference may not be much relevant in $D = 5$ dimensions: the reason being that an arbitrary metric $g_{55}(y)$ times an arbitrary power of the warp factor is still an arbitrary metric. However, in dimensions $D \geq 6$, the choice of γ would be relevant since it ought to be related to the Ricci curvature of the internal manifold as explicitly shown in [17]; we just need to relate the coefficient γ to $R^{(m)}$. Especially, for the discussion of no-go theorems in [10, 12, 15], the choice γ is not very important, for the theorems of these papers ruled out the existence of de Sitter solutions in pure supergravity just because of an extra condition on the warp factor, so-called the boundedness condition $\int \nabla^2 W^4 = \int \left(W^4\right)'' - 2\gamma \int W^2 W'^2 = 0$, which is, however, not satisfied by cosmological solutions, especially, when the extra dimensions are only geometrically compact and/or when there are localized sources like branes and orientifold planes. In our analysis below we shall relax the condition like $\int \nabla^2 W^n = 0$ (where n is some constant) until we are ready to comment on this part of the problem.

One could naively think that the coefficient γ plays no role in the discussion of warped compactifications. The reason is that, since the metric $g_{mn}(y)$ is arbitrary, an arbitrary metric times an arbitrary power of the warp factor is still an arbitrary metric. Here one should also note that the metric $g_{mn}(y)$ is *not* just a single canonical function of y but has more than one components, $(m,n) = 1, 2, \cdots (D-4)$. In dimensions $D \geq 6$, one cannot absorb W^γ into $g_{mn}(y)$ just by using some coordinate transformations unless that each and every components of $g_{mn}(y)$ are equal or proportional to the same function, say $f(y)$. For clarity, let us take $D = 10$ and write the 6d metric as

$$ds^2_6 = h(y)\, dy^2 + f(y)\, \tilde{g}_{mn} d\Theta^m d\Theta^n, \qquad (2)$$

where $\tilde{g}_{mn}$ denote the metric components of the five-dimensional base space X_5, which are independent of the y coordinate. The volume factor W^γ in Eq. (1) may be absorbed inside dy^2 by using the transformation $W^\gamma h(y) dy^2 \equiv d\tilde{y}^2$ and also defining a new function $X(\tilde{y})$ such that $W^\gamma f(y) \equiv X(\tilde{y})$. With these substitutions, the warp factor W^2 multiplying the 4d part of the metric is $[X(\tilde{y})/f(\tilde{y})]^{2/\gamma}$. The 10D metric still involves two unknown functions and the free parameter γ. That is to say, if we want to write a general metric ansatz (for the purpose of solving Einstein's equations), then we have to allow one more free parameter in the metric than that were considered in [10, 12, 15].

It is not difficult to check that only a specific value of γ would give a nontrivial cosmological solution, once we specify the 6d metric or fix the spatial curvature of the internal space. Suppose we chose $\gamma = 0$ and then simultaneously assumed that the internal space is Ricci flat, then we would not find a de Sitter solution at least in pure supergravity. The story would be similar for some other specific choices of γ and/or the internal curvature. For example, if we set $\gamma = -2$ in (1), then we would find a de Sitter solution only by allowing Y_6 to have negative curvature. In view of this discussion, at this stage we shall keep both the coefficient γ and the curvature of the internal space arbitrary.

II. AN EXPLICIT MODEL IN $D = 5$ DIMENSIONS

Let us first consider a specific example where the real world looks like a five-dimensional universe described by the metric

$$ds^2_5 = W(y)^2\, \hat{g}_{\mu\nu} dX^\mu dX^\nu + \rho^2\, W(y)^\gamma\, dy^2, \qquad (3)$$

where ρ is the radius of compactification, which may be assumed to be a constant in the simplest scenario under consideration. The classical action describing this warped geometry is given by

$$S = \frac{M_5^3}{2} \int d^5x \sqrt{-g_5}\, R_{(5)}, \qquad (4)$$

where M_5 is the fundamental 5D Planck scale. Our starting point is different from that in the RS braneworld models only in that we take the metric of the usual four-dimensional spacetime in a general form

$$\begin{aligned} ds^2_4 &= -dt^2 + a^2(t) \left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right] \\ &\equiv \hat{g}_{\mu\nu} dX^\mu dX^\nu, \end{aligned} \qquad (5)$$

where $a(t)$ is the scale factor of the universe. We have allowed all three possibilities for the physical 3D spatial curvature: flat ($k = 0$), open ($k < 0$) and closed ($k > 0$). Models similar to the one here were studied before, see for example [18, 19], but an interesting (and perhaps new) observation is that for the existence of inflationary de Sitter solutions we do not necessarily require a 5D bulk cosmological constant term.

The three independent Einstein's equations following from the metrics (3) and (5) are given by

$$W'^2 - \rho^2\left(\frac{\dot{a}^2}{a^2}+\frac{k}{a^2}\right)W^\gamma = 0, \tag{6a}$$

$$2WW'' - \gamma W'^2 = 0, \tag{6b}$$

$$\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2} - \frac{k}{a^2} = 0, \tag{6c}$$

where $\dot{}$ and $'$ denote respectively $\partial/\partial t$ and $\partial/\partial y$ (or $\partial/\partial z$ when $\gamma = 2$). From Eq. (6c) we immediately obtain

$$a(t) = \frac{1}{2}\exp\left(\frac{\mu(t-t_0)}{\rho}\right) + \frac{k\,\rho^2}{2\mu^2}\exp\left(\frac{\mu(t_0-t)}{\rho}\right), \tag{7}$$

where μ and t_0 are integration constants. In the $\gamma \neq 2$ case, from Eqs. (6a) and (6b), we obtain

$$[W(y)]^{2-\gamma} = \frac{1}{4}(2-\gamma)^2\mu^2(y+c)^2. \tag{8}$$

The bulk singularity at $y = -c$ is just a coordinate artifact, which could simply be absent in some other coordinate systems. To quantify this we can either introduce a new coordinate z satisfying $W(y)^{\gamma/2}dy \equiv W(z)\,dz$ or solve the 5D Einstein equations by setting $\gamma = 2$ in (3) and replacing y there by z. We then get

$$ds_5^2 = e^{-2\mu z}\left(\hat{g}_{\mu\nu}dX^\mu dX^\nu + \rho^2\,dz^2\right). \tag{9}$$

One could in principle set $\rho = 1$ in Eq. (9) or in Eq. (7), but an essential point here is that the scale factor and warp factor can have quite different slopes. In natural Planck's unit one may require $\rho \ll 1$ (see below). Note that, with $\mu > 0$, the universe must accelerate eventually. In a sense the universe accelerates due to a kind of back reaction of the 5D warped geometry on the usual four-dimensional spacetime.

For the above solution the 4D effective Newton's constant is not finite. The reason being that in (9) z ranges from $-\infty$ to $+\infty$, and hence the extra dimension has infinite warped volume. In order to get physical results, including a finite 4D Newton's constant, we shall introduce some elements of RS type braneworld models.

A. A geometrically compact extra dimension

To this end, we specify a boundary condition such that the warp factor is regular at $z = 0$ where we place a 3-brane with brane tension T_3. We also introduce a bulk cosmological term Λ. The classical action describing this set up is

$$S = \frac{M_5^3}{2}\int d^5x\sqrt{-g_5}\,(R-2\Lambda) + \frac{M_5^3}{2}\int d^4x\sqrt{-g_b}(-T_3), \tag{10}$$

where g_b is the determinant of the metric g_{ab} evaluated at $z = 0$. Einstein's equations are given by

$$G_{AB} = -\frac{T_3}{2}\frac{\sqrt{-g_b}}{\sqrt{-g}}g^b_{\mu\nu}\delta^\mu_A\delta^\nu_B\delta(z) - \Lambda g_{AB}. \tag{11}$$

Eqs. (6a) and (6b) get modified as

$$W'^2 - \rho^2\left(\frac{\dot{a}^2}{a^2}+\frac{k}{a^2}\right)W^\gamma = -\frac{\hat{\Lambda}}{6}W^{2+\gamma}, \tag{12a}$$

$$2WW'' - \gamma W'^2 = -\frac{\hat{\Lambda}}{3}W^{2+\gamma} - \frac{\tau_3\,\delta(z)}{3}W^{2-\gamma/2}, \tag{12b}$$

where $\hat{\Lambda} \equiv \Lambda\,\rho^2$ and $\tau_3 \equiv T_3\,\rho^2$, while Eq. (6c) is the same, which is unaffected by a bulk cosmological term. One replaces $\delta(z)$ by $\delta(y-y_0)$ in the $\gamma \neq 2$ case. With the widely used choice that $\gamma = 0$, we get [20]

$$W(y) = \frac{\sqrt{6}}{\mu\sqrt{-\hat{\Lambda}}}\sinh\left[\frac{\sqrt{-\hat{\Lambda}}\,(y+c)}{\sqrt{6}}\right]. \tag{13}$$

By defining $W(y)^{\gamma/2}dy \equiv W(z)\,dz$, we obtain

$$W(z) = \frac{24\mu^2}{24\mu^2 e^{\mu|z|} + \Lambda\rho^2\,e^{-\mu|z|}}, \tag{14}$$

which has a smooth $\Lambda \to 0$ limit. This result is nothing but an exact solution of 5D Einstein equations with $\gamma = 2$ in Eq. (3). In the above we demanded a Z_2 symmetry about the brane's position at $z = 0$. If we relax this symmetry, then the warp factor becomes singular at $z = -\frac{1}{2\mu}\ln\left(-\frac{24\mu^2}{\Lambda\rho^2}\right)$, especially, with $\Lambda < 0$. We shall therefore consistently demand a Z_2 symmetry about the brane's position at $z = 0$, irrespective of the choice $\Lambda = 0$ or $\Lambda < 0$. It is not difficult to check that Einstein's equations are satisfied at $z = 0$ when

$$T_3 = \frac{24\mu^2 - \Lambda\rho^2}{2\mu\rho^2}. \tag{15}$$

The brane tension is positive when $\mu^2 > -\Lambda\rho^2/24$. As in RS models [8, 9], the choice $\Lambda < 0$ could be more physical.

The solution (14) is defined up to a rescaling of z coordinate, implying that

$$W(z) = \frac{24\mu^2}{24\mu^2\,e^{\mu(|z|+z_0)} + \Lambda\rho^2\,e^{-\mu(|z|+z_0)}}. \tag{16}$$

In the $\Lambda = 0$ case, we take $z_0 = 0$ so that $W(z) = 1$ at $z = 0$. In the $\Lambda < 0$ case, we take

$$e^{\mu z_0} = \frac{1}{2} + \frac{1}{2}\sqrt{1 - \frac{\Lambda\rho^2}{6\mu^2}}. \tag{17}$$

In the limit $\mu \to 0$, we get $W(z) \to 1$ and $a(t) \to$ const (especially when $k = 0$), giving rise to a 5D Minkowski

or AdS$_5$ spacetime depending on the choice that $\Lambda = 0$ or $\Lambda < 0$.

The four-dimensional effective theory follows by substituting Eq. (3) into the classical action (10). Here we focus on the 5D curvature term from which we can derive the scale of gravitational interactions:

$$S_{\rm eff} \supset \frac{M_5^3\,\rho}{2}\int d^4x\sqrt{-\hat{g}_4}\int dz\, W^{2+\gamma/2} \times\left(\hat{R}_4 - \mathcal{L}_0 - 2\Lambda W^2\right), \tag{18}$$

where $\mathcal{L}_0 \equiv \rho^{-2}\,W^{-\gamma}\left(12W'^2 + 8WW'' - 4\gamma W'^2\right)$. As a simple example, henceforth we take $\gamma = 2$. Hence

$$\mathcal{L}_0 = \frac{12\mu^2}{\rho^2}\left(1 - \frac{160\Lambda\mu^2\rho^2}{(24\mu^2\,e^{\mu|z|} + \Lambda\rho^2\,e^{-\mu|z|})^2}\right) - \frac{16\mu}{\rho^2}\left(\frac{24\mu^2 e^{\mu|z|} - \Lambda\rho^2 e^{-\mu|z|}}{24\mu^2 e^{\mu|z|} + \Lambda\rho^2 e^{-\mu|z|}}\right)\delta(z). \tag{19}$$

This result shows that a negative bulk cosmological term could make the value of 4D effective cosmological constant more positive. In the particular case that $\Lambda = 0$, the above expression takes a much simpler form

$$\mathcal{L}_0 = \rho^{-2}\left(12\mu^2 - 16\mu\delta(z)\right). \tag{20}$$

The relation between four- and five-dimensional effective Planck masses is then given by

$$M_{\rm Pl}^2 = M_5^3\,\rho\int_{-\infty}^{\infty} dz\, e^{-3\mu|z|} = \frac{2M_5^3\rho}{3\mu}. \tag{21}$$

In the limit $\mu \to 0$, the extra dimension z opens up and we thus obtain a 5D Minkowski space ($M_{\rm Pl}^2 \to \infty$ or $G_4 \to 0$). However, in the generic situation with $\mu > 0$, the 4D Newton's constant is finite.

Although the details and the motivations are different, the $\Lambda = 0$ solution above bears certain features of a 5D braneworld model discussed by Dvali et al. [21] where a cosmic (self-)acceleration of the universe is supported by the 4D scalar curvature term on the brane. In the present approach, however, the 5D spacetime is non-factorizable and the universe accelerates because of a positive curvature ($R_4 > 0$) induced by the 5D warped geometry.

B. A physically compact extra dimension

The above analysis can easily be extended to a set up with two 3-branes, as in RS1 braneworld model. To this end, one introduces a 5D bulk cosmological term Λ and also specifies boundary conditions such that the warp factor is regular both at orbifold fixed points $y = 0$ and $y = \pi$ where we place two 3-branes (b_1 and b_2) with brane tension $T_3^{(1)}$ and $T_3^{(2)}$, respectively. We start with a canonical metric (choosing $\gamma = 0$ in (3))

$$ds_5^2 = W(y)^2\,\hat{g}_{\mu\nu}dX^\mu dX^\nu + \rho^2\,dy^2, \tag{22}$$

where as above $0 \le y \le \pi$ is the coordinate for an extra dimension, which is a finite interval whose size is set by ρ.

The classical action describing this set up is

$$S = \frac{M_5^3}{2}\Big(\int d^5x\sqrt{-g_5}\,(R - 2\Lambda) + \int d^4x\sqrt{-g_{\rm b1}}(-T_3^{(1)}) + \int d^4x\sqrt{-g_{\rm b2}}(-T_3^{(2)})\Big), \tag{23}$$

where g_{b1} and g_{b2} are determinants of the metric g_{ab} evaluated at $y = \pi$ and $y = 0$. The 5D Einstein equations read

$$\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2} - \frac{k}{a^2} = 0, \tag{24a}$$

$$\frac{W'^2}{W^2} - \frac{\rho^2}{W^2}\left(\frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right) + \frac{\Lambda\rho^2}{6} = 0, \tag{24b}$$

$$\frac{W''}{W} + \frac{\rho T_3^{(1)}}{6}\,\delta(y-\pi) + \frac{\rho T_3^{(2)}}{6}\,\delta(y) + \frac{\Lambda\rho^2}{6} = 0, \tag{24c}$$

The solution to Eqs. (24a)-(24b) consistent with the orbifold symmetry $y \to -y$ is

$$a(t) = \frac{1}{2}\exp\left(\frac{\mu t}{\rho}\right) + \frac{k\,\rho^2}{2\mu^2}\exp\left(-\frac{\mu t}{\rho}\right), \tag{25a}$$

$$W(y) = \frac{\sqrt{6}}{\mu\sqrt{-\Lambda\rho^2}}\sinh\left[\frac{\sqrt{-\Lambda\rho^2}}{\sqrt{6}}\,(|y| - y_0)\right]. \tag{25b}$$

Note that, in computing derivatives of W, we are to consider the metric a periodic function in y. Eq. (25b), valid for $-\pi \le y \le \pi$, then implies

$$\frac{W''}{W} + \frac{\Lambda\rho^2}{6} + \sqrt{\frac{-\Lambda\rho^2}{6}}\coth\left(\sqrt{\frac{-\Lambda\rho^2}{6}}(|y| - y_0)\right) \times\left[2\delta(y-\pi) - 2\delta(y)\right] = 0. \tag{26}$$

Note that, unlike in RS1 brane world model, we do not necessarily require $T_3^{(1)} = -T_3^{(2)}$; the brane tensions could well depend on their positions. By placing them at $y = \pi$ and $y = 0$, from Eqs. (24c) and (26) we find

$$T_3^{(1)} = 2\sqrt{-6\Lambda\rho^2}\coth\left(\sqrt{\frac{-\Lambda\rho^2}{6}}(\pi - y_0)\right), \tag{27a}$$

$$T_3^{(2)} = -\,2\sqrt{-6\Lambda\rho^2}\coth\left(\sqrt{\frac{-\Lambda\rho^2}{6}}(-\,y_0)\right). \tag{27b}$$

By defining $\Lambda \equiv -6/L^2$, where L is the curvature length associated with AdS$_5$ space, we get

$$W(y) = \frac{L}{\rho\mu}\sinh\left(\frac{\rho}{L}(|y| - y_0)\right). \tag{28}$$

The bulk singularity at $|y| = y_0$ may be avoided by taking $y_0 < 0$, in which case one of the 3-branes would have a negative tension. The Goldberger and Wise mechanism to stabilize the size of fifth dimension or radion using a nontrivial bulk scalar field [22] may be applied to the present model, but in this Letter we do not study such effect.

III. REVISITING BRANEWORLD NO-GO THEOREMS

The no-go theorems of [10, 11, 12] claim that vacuum solutions of the type presented above should not exist, while we have explicitly shown the existence of a four-dimensional de Sitter solution within 5D Einstein gravity. There arises an important question as: What prevented the previous authors from inventing (or ruling out) the explicit de Sitter solutions given above? To answer this question we need to carefully examine the conditions embedded in the discussion of the earlier no-go theorems. Below we will focus on the case of a 5D Minkowski bulk, but its generalization in higher dimensions should be straightforward.

A. No-go theorem in five-dimensions

For the metric (3), the basic equations reduce to

$$^{(5)}R_{\mu\nu} = {}^{(4)}R_{\mu\nu} - \frac{\hat{g}_{\mu\nu}}{4W^\gamma}\left[\frac{\left(W^4\right)''}{W^2} - 2\gamma W'^2\right], \tag{29a}$$

$$R_{55} = -\frac{4}{W}W'' + \frac{2\gamma}{W^2}W'^2. \tag{29b}$$

Here, for simplicity, we have set $\rho = 1$. We may rewrite the above two equations as follows

$$R_g = R_{\hat{g}}\, W^{-2} - 2(6-\gamma)W'^2W^{-2-\gamma} - 4W''W^{-1-\gamma}, \tag{30a}$$

$$R_5{}^5 = -4W''W^{-1-\gamma} + 2\gamma W'^2W^{-2-\gamma}, \tag{30b}$$

where $R_g \equiv {}^{(5)}R_\mu{}^\mu$ and $R_{\hat{g}} \equiv {}^{(4)}R_\mu{}^\mu$ are, respectively, the curvature scalars of the 5- and 4-dimensional spacetimes with the metric tensors $g_{\mu\nu}$ and $\hat{g}_{\mu\nu}$. A linear combination of $(1-n)W^{n+\gamma}\times$ Eq. (30a) and $(n-4)W^{n+\gamma}\times$ Eq. (30b) gives (where n is an arbitrary constant)

$$\frac{(W^n)''}{n} - \frac{\gamma}{2}W'^2W^{n-2} = \left[\frac{1-n}{12}\left(R_g - R_{\hat{g}}W^{-2}\right) + \frac{n-4}{12}R_5{}^5\right]W^{n+\gamma}. \tag{31}$$

From the 5D Einstein equations

$$R_A^B = 8\pi G_5\left(T_A^B - \frac{1}{3}\,\delta_A^B T_C{}^C\right),$$

we obtain

$$^{(5)}R_\mu^\mu = 8\pi G\left(-\frac{1}{3}\,{}^{(5)}T_\mu^\mu - \frac{4}{3}\,T_5^5\right)$$

and

$$R_5^5 = 8\pi G\left(-\frac{1}{3}\,{}^{(5)}T_\mu^\mu + \frac{2}{3}\,T_5^5\right).$$

From Eq. (31) we then find

$$\left(A'e^{nA}\right)' - \frac{\gamma}{2}A'^2e^{nA} + \frac{1-n}{12}\,R_{\hat{g}}\,e^{(n+\gamma-2)A} = \frac{2\pi G_5}{3}\left(T_g + (2n-4)T_5{}^5\right)e^{(n+\gamma)A}, \tag{32}$$

where $e^{A(y)} \equiv W(y)$ and $T_g \equiv {}^{(5)}T_\mu{}^\mu$. With $\gamma = 0$, we recover the braneworld sum rule discussed in [23].

We argue that the warp factor constraints such as $\oint \nabla^2 W^4 = 0$ and $\oint \nabla(W^{n-1}\nabla W) = 0$ discussed in [10, 24] are 'strict', which are not essentially satisfied by cosmological solutions, especially, when the extra dimensions are only geometrically compact. For clarity, take $\gamma = 2$ and thus $W(z) = e^{A(z)} = e^{-\mu|z|}$. We then find

$$\begin{aligned}\oint \nabla^2 W^4 &\equiv \oint \left(W^4\right)'' - 2\gamma\oint W'^2W^2 \\ &= 4\oint W^3W'' + 8\oint W^2W'^2 \\ &= \oint e^{-4\mu|z|}\left(12\mu^2 - 8\mu\,\delta(z)\right) \neq 0.\end{aligned} \tag{33}$$

There can be an additional condition on the warp factor, i.e. the finiteness of 5D warped volume $1/G_N \sim \int W^{2+\gamma/2}$ =const. This holds in the above example because of a Z_2 symmetry under $z \to -z$.

Coming back to Eq. (32), and following [23], let us assume that there exists a class of solutions for which $\oint \left(A'e^{nA}\right)' = 0$, which is plausible if the extra dimension is like a closed cycle or compact. We then find

$$\oint \left(T_g + (2n-4)T_5{}^5\right)e^{(n+\gamma)A} = \frac{1-n}{8\pi G_5}R_{\hat{g}}\oint e^{(n+\gamma-2)A} - \frac{3\gamma}{4\pi G_5}\oint A'^2\mathrm{e}^{nA}.$$

We can get $R_{\hat{g}} > 0$ by appropriately choosing n or γ, even if the term on the left-hand side vanishes. This result is consistent with some explicit de Sitter solutions of 5D Einstein equations presented above (cf. Eqs. (7)-(9)).

In the presence of a bulk cosmological constant Λ, the 5D energy momentum tensor is given by

$$T_{AB} = -\frac{1}{8\pi G_5}\left(\Lambda g_{AB} + \frac{T_3}{2}\,\delta(z)P(g_{AB})\right), \tag{34}$$

where $P(g_{AB}) \equiv g_{\mu\nu}\delta_A^\mu\delta_B^\nu/\sqrt{g_{zz}}$. From this we derive

$$T_g + (2n-4)T_5{}^5 = \frac{-2n\Lambda - 2T_3\delta(z)}{8\pi G_5}. \tag{35}$$

By demanding that $\Lambda < 0$, and with a suitable choice of n or γ, we can obtain a de Sitter solution, i.e. $R_{\hat{g}} > 0$ even if the brane tension is positive. In the case $T_3 < 0$, the cosmic acceleration of a four-dimensional universe seems more plausible due to an explicit violation of 4D strong energy condition, but the choice $T_3 < 0$ is not well motivated (at least in a single brane set up).

In summary, our results above show that if we do not enforce the warp factor constraint such as $\int \nabla^2 W^4 = 0$, which does not hold in several examples considered in this Letter, then it is possible to realize a cosmological de Sitter solution even within some simplest or canonical warped braneworld and supergravity models.

B. No-go theorem in ten-dimensions

In spacetime dimensions $D \geq 6$ (or $m \geq 2$), with a judicious choice of γ, we can find de Sitter solutions with all three different choices of the internal curvature, i.e. $R^{(m)} = 0$, $R^{(m)} > 0$ and $R^{(m)} < 0$. This could again be seen in contrast to the no-go theorems discussed in [10, 12]. We should therefore have a closer look on the earlier no-go arguments. Assuming that ten-dimensional supergravity is the relevant framework, we may write the 10D metric as

$$ds_{10}^2 = e^{2A(y)} ds_4^2 + \rho^2 \, e^{\gamma A(y)} ds_6^2 \tag{36}$$

with

$$ds_6^2 = dy^2 + dy_1^2 + \cdots + dy_5^2, \quad (\epsilon = 0) \tag{37a}$$

$$ds_6^2 = dy^2 + \sin^2 y \, d\Omega_5^2, \quad (\epsilon = +1) \tag{37b}$$

$$ds_6^2 = dy^2 + \sinh^2 y \, d\Omega_5^2, \quad (\epsilon = -1), \tag{37c}$$

where $d\Omega_5^2$ represents the metric of a usual 5-sphere. In the above example, the internal 6d manifold is maximally symmetric, $\tilde{R}_{mn} = \epsilon(m-1)\tilde{g}_{mn}$. A straightforward calculation gives

$${}^{(10)}R_{\mu\nu} = {}^{(4)}\hat{R}_{\mu\nu} - \frac{\hat{g}_{\mu\nu}}{\rho^2}\left(\nabla_y^2 A + 2(2+\gamma)A'^2\right) e^{(2-\gamma)A}, \tag{38}$$

$$\begin{aligned} {}^{(10)}R_{mn}(x,y) &= {}^{(6)}\tilde{R}_{mn} - 2(2+\gamma)A'^2 \tilde{g}_{mn}^{(6)} \\ &\quad -\frac{\gamma}{2}\tilde{g}_{mn}^{(6)} \nabla_y^2 A - 2(3+\gamma)\nabla_m \partial_n A \\ &\quad - \left(8 - (2+\gamma)^2\right) \partial_m A \partial_n A, \end{aligned} \tag{39}$$

where

$$\nabla_y^2 A = \begin{cases} A'', \\ A'' + 10A' \cot y, \\ A'' + 10A' \coth y, \end{cases} \tag{40}$$

respectively, for the metrics (37a), (37b) and (37c). In the particular case that $\gamma = 2$, we get [12]

$$\begin{aligned} {}^{(10)}R_{\mu\nu} &= {}^{(4)}\hat{R}_{\mu\nu} - \frac{\hat{g}_{\mu\nu}}{\rho^2}\left(\nabla_y^2 A + 8A'^2\right) \\ &= {}^{(4)}\hat{R}_{\mu\nu} - \hat{g}_{\mu\nu} \frac{e^{-8A(y)}}{8\rho^2} \nabla_y^2 e^{8A(y)}. \end{aligned} \tag{41}$$

On the other hand, from the 10D Einstein equations $R_{AB} - (1/2) g_{AB} R = 8\pi G_{10} T_{AB}$, we obtain

$$\begin{aligned} {}^{(10)}R_{\mu\nu} &= 8\pi G_{10}\left(T_{\mu\nu} - \frac{1}{8} e^{2A} \hat{g}_{\mu\nu} T_C^C\right), \\ {}^{(10)}R_\mu^\mu &= 4\pi G_{10}\left(T_\mu^\mu - T_m^m\right). \end{aligned} \tag{42}$$

The above result shows that, with $\int \nabla_y^2 e^{8A(y)} = 0$, a de Sitter spacetime (with ${}^{(4)}\hat{R} > 0$) cannot be realized without sources of T_{00} which violate the positive energy condition, i.e. without violating the condition $T_m^m - T_\mu^\mu \geq 0$ (see also the discussion below Eq. (2.15) in [15]). This is the no-go theorem of Maldacena and Nunez [12].

The above discussion is special at least from two aspects. First, it only covered the case $\gamma = 2$, for which $e^{(2-\gamma)A} = 1$ for any choice of $A(y)$. Second, the condition on the warp factor, i.e. $\int \nabla_y^2 e^{8A(y)} = 0$ is 'strict' and it is not always satisfied, especially, in the presence of some brane sources. Further, the 6d metric of the form (37a)-(37c) was not sufficiently general as it contained no free parameter that can be tuned or fixed according to the choice of γ in the warp factor. By relaxing the condition like $\oint \nabla_y^2 e^{nA} = 0$ (where n is some constant) or some other similar constraints one should expect de Sitter solution to be easy to realize. Below we will give a couple of explicit examples.

IV. AN EXPLICIT MODEL IN $D = 10$ DIMENSIONS

Let us generalize the 6d metric in Eq. (37a)-(37c) as follows

$$ds_6^2 = g_{mn}^{(6)}(y) \, dy^m dy^n \equiv h(y) \, dy^2 + f(y) \, ds_{X_5}^2, \tag{43}$$

where $h(y)$ and $f(y)$ are two arbitrary functions. X_5 can be taken to be a usual five-sphere S^5 or some other compact Einstein manifolds. One of the well motivated examples is the Einstein-Sasaki space $(S^2 \times S^2) \rtimes S^1$ with metric [25]

$$ds_{X_5}^2 = \frac{1}{6}\left(e_{\theta_1}^2 + e_{\phi_1}^2 + e_{\theta_2}^2 + e_{\phi_2}^2\right) + \frac{1}{9} e_\psi^2, \tag{44}$$

where $e_{\theta_i} = d\theta_i$, $e_{\phi_i} = \sin\theta_i d\phi_i$ and $e_\psi \equiv d\psi + \cos\theta_1 d\phi_1 + \cos\theta_2 d\phi_2$, (θ_1, ϕ_1) and (θ_2, ϕ_2) are coordinates on each S^2 and ψ is the coordinate of a $U(1)$ fiber. One could in principle start with six-dimensional deformed conifold metrics without any conical singularities, such as in [26, 27], or a deformed six-sphere as considered in [28], but we find the metric ansatz (43) sufficiently simple for the purpose of solving 10D Einstein equations analytically. For the Ansätze (36) and (43), a straightforward calculation gives

$$\begin{aligned} &{}^{(10)}R_{\mu\nu}(x,y) \\ &= {}^{(4)}\hat{R}_{\mu\nu} - \frac{\hat{g}_{\mu\nu} e^{(2-\gamma)A}}{\rho^2 h}\left(\nabla_y^2 A + 2(2+\gamma)A'^2\right) \\ &= {}^{(4)}\hat{R}_{\mu\nu}(x) - \frac{\hat{g}_{\mu\nu} \, e^{(2-\gamma)A}}{\rho^2 h} \\ &\quad \times \left[A'' + \frac{1}{2}\left(\frac{5f'}{f} - \frac{h'}{h}\right) A' + 2(2+\gamma)A'^2\right]. \end{aligned} \tag{45}$$

$$R_{yy} = -\frac{8+5\gamma}{2}A'' - 2(2-\gamma)A'^2 + \frac{(8+5\gamma)h'A'}{4h} - \frac{5\gamma f'A'}{4f} + \frac{5}{4}\left(\frac{f'^2}{f^2} + \frac{h'f'}{hf} - \frac{2f''}{f}\right), \quad (46)$$

$$^{(10)}R_{pq} = {}^{(6)}R_{pq} - \tilde{g}_{pq}\Big(\frac{\gamma f A''}{2h} + \frac{\gamma(2+\gamma)fA'^2}{h} + \frac{(8+9\gamma)f'A'}{4h} - \frac{\gamma f h'A'}{4h^2}\Big), \quad (47)$$

where $' \equiv \partial/\partial y$ and

$$^{(6)}R_{pq} = \left(4 - \frac{3f'^2}{4hf} - \frac{f''}{2h} + \frac{h'f'}{4h^2}\right)\tilde{g}_{pq}. \quad (48)$$

In the above $\tilde{g}_{pq}$ denote the metric components of the base space X_5, which are independent of the y coordinate.

Example 1. Assume that $h(y) = 1$ and $f(y) \equiv \alpha_1(y-y_0)^2$. The 6d metric takes the form

$$ds_6^2 = g^{(6)}_{mn}(y)\,dy^m dy^n \equiv dy^2 + \alpha_1(y-y_0)^2\,ds^2_{X_5}. \quad (49)$$

With $y_0 = 0$, y measures the radius of the base space X_5, so the coordinate range is $0 \le y \le \infty$. Especially, when $\alpha_1 \neq 1$, the metric (49) is singular at $y = y_0$. This leads to an undesirable result that the warp factor $e^{A(y)}$ vanishes at $y = y_0$. To see this one can solve the 10D Einstein equations explicitly. The solution is given by

$$e^{A(y)} = \left(\frac{3\mu^2(2-\gamma)^2(y-y_0)^2}{32}\right)^{1/(2-\gamma)}, \qquad \alpha_1 \equiv \frac{(2-\gamma)^2}{8}, \quad (50)$$

with the same scale factor as given in (7). This yields

$$\nabla_y^2 A = \frac{8}{(2-\gamma)(y-y_0)^2} + \frac{4\delta(y)}{(2-\gamma)(y-y_0)}. \quad (51)$$

As is evident, this solution does not satisfy the constraint like $\oint \nabla_y^2 A = 0$ or $\oint \nabla_y^2 e^{nA(y)} = 0$.

Example 2. Assume that $h(y) = \sinh^2(y-y_0)$ and $f(y) \equiv \alpha_1 \cosh^2(y-y_0)$. The 6d metric is

$$ds_6^2 = \sinh^2(y-y_0)\,dy^2 + \alpha_1\cosh^2(y-y_0)\,ds^2_{X_5}. \quad (52)$$

With $\alpha_1 \equiv (2-\gamma)^2/8$, the 10D Einstein equations are explicitly solved for

$$e^{A(y)} = \left(\frac{3\mu^2(2-\gamma)^2\cosh^2(y-y_0)}{32}\right)^{1/(2-\gamma)}. \quad (53)$$

The 10d metric solution is given by

$$\begin{aligned} ds^2_{10} &= e^{2A(y)}\left(ds_4^2 + \frac{32\rho^2\tanh^2(y-y_0)}{3\mu^2(2-\gamma)^2} \times\left(dy^2 + \frac{(2-\gamma)^2}{8}\coth^2(y-y_0)\,ds^2_{X_5}\right)\right) \\ &\propto u^{4/(2-\gamma)}\left(ds_4^2 + \frac{32\rho^2}{3\mu^2(2-\gamma)^2u^2} \times\left(du^2 + \frac{(2-\gamma)^2}{8}u^2\,ds^2_{X_5}\right)\right), \end{aligned} \quad (54)$$

where $u \equiv \cosh(y-y_0) \equiv \sqrt{32/3\mu^2(2-\gamma)^2}\,e^{(2-\gamma)A/2}$. This gives

$$\nabla_y^2 A = \frac{8\tanh^2(y-y_0)}{(2-\gamma)} + \frac{4\tanh(y-y_0)\,\delta(y)}{(2-\gamma)}. \quad (55)$$

From Eq. (45) we then obtain

$$^{(10)}R_{\mu\nu} = {}^{(4)}\hat{R}_{\mu\nu} - \frac{\hat{g}_{\mu\nu}}{\rho^2}\left(3\mu^2 + \frac{3\mu^2(2-\gamma)}{8}\coth(y-y_0)\,\delta(y)\right). \quad (56)$$

The exact solution above violates the warp factor constraint like $\int \nabla_y^2 e^{nA} = 0$. Moreover, the 6d warped volume is not constant. Rather it scales as $V_6^{\rm w} \sim \int d\Omega_5 \int e^{(2+3\gamma)A}\sqrt{g_6}\,dy \sim \int u^{(14+\gamma)/(2-\gamma)}du$. Although one can hope to get an ideal situation with almost constant or slowly varying warped factor, for instance, by invoking some non-perturbative effects (as in KKLT model [16]) or introducing certain α' corrections, the solution above is interesting is the regard that the radius modulus, which scales as $|\tanh y|$, is constant in the limit $y \to \infty$ [17]. Further, unlike with some singular conifold metrics considered in the literature, for instance [26, 29], our solution is regular everywhere.

Coming back to the metric (49), it is not difficult to see that the singularity of this metric at $y = y_0$ (especially, when $\alpha_1 \neq 1$) is just a coordinate artifact. To quantify this, we may introduce a new coordinate z, which is related to the usual coordinate y via $y \propto e^{-\lambda z}$ (where λ is some constant). The 10D metric that explicitly solves all of the Einstein equations and is consistent with Z_2 symmetry ($z \to -z$) about the brane's position ($z = 0$) is

$$\begin{aligned} ds^2_{10} &= e^{2A(z)}\left(ds_4^2 + \frac{8\rho^2}{3\mu^2\,\ell^2}\,dz^2 + \frac{4\rho^2}{3\mu^2}\,ds^2_{X_5}\right), \\ A(z) &= -\frac{|z|}{\ell} - \frac{A_0}{2}. \end{aligned} \quad (57)$$

From this, we derive $\nabla_z^2 A = -(2/\ell)\delta(z)$ and hence

$$^{(10)}R_{\mu\nu} = {}^{(4)}\hat{R}_{\mu\nu} - \frac{3\mu^2\hat{g}_{\mu\nu}}{8\rho^2}\left(-\frac{2}{\ell}\,\delta(z) + \frac{8}{\ell^2}\right). \quad (58)$$

If we do not enforce the Z_2 symmetry, then the solution above satisfies $\nabla_z^2 A = 0$. In this case, a four-dimensional de Sitter solution with ${}^{(4)}\hat{R}_{00} < 0$ is still possible, but there arises an important difference: since z ranges from $-\infty$ to $+\infty$, the 6d warped volume can be arbitrarily large. Typically, $V_6^{\rm w} \sim \int e^{8A}\sqrt{\tilde{g}_6} \sim \frac{64\sqrt{2}}{27}\, e^{-4A_0} \int d\Omega_5 \int dz\, e^{-(8/\ell)z}$, where $\int d\Omega_5 = \frac{1}{108}\int d(\cos\theta_1) d(\cos\theta_2) d\phi_1 d\phi_2 d\psi = 16\pi^3/27$. To get a sensible result with an almost constant warped volume (or slowly varying warp factor), we need to send $\ell \to \infty$ or take $e^{-4A_0} \to 0$.

The metric solution (54) is already regular everywhere, but we may introduce some brane sources at $y = y_0$ and then solve the 10d Einstein equations with proper regularity conditions at $y = y_0$. This was in fact done quite recently in the second paper in [17], so in the following discussion we only consider the metric solution (57).

To solve the Einstein equations at $z = 0$, we shall write

$$G_A^B = \tau_p\, P(g_A^B), \tag{59}$$

where $P(g_A^B)$ is the pull-back of the spacetime to the world volume of the p-brane ($3 \le p \le 8$) with tension τ_p. We shall impose a Z_2 symmetry at the brane's position at $z = 0$. The warp factor will then have a discontinuity in its first derivative, implying that $\partial|z|/\partial z = \mathrm{sgn}(z)$ and $\partial^2|z|/\partial z^2 = 2\delta(z)$. We then obtain

$$G_z^z|_{z=0} = 0, \quad G_M^N|_{z=0} = \frac{6\mu^2\ell}{\rho^2}\, e^{A_0}\, \delta(0) \eta_M^N, \tag{60}$$

where $(M, N) = t, x_i, \theta_i, \phi_i, \psi$. We only consider the simplest case that $p = 8$, for which the brane extends to all of the dimensions except along the z-direction and thus $P(g_A^B) = \delta(z)/\sqrt{g_{zz}}$. Einstein's equations are satisfied at $z = 0$ when

$$\tau_8 = \frac{4\sqrt{6}\,\mu}{\rho}. \tag{61}$$

From Eq. (57) we derive

$$\begin{aligned} M_{Pl}^2 &= \frac{M_{10}^8}{(2\pi)^6} \frac{32\sqrt{2}\,\rho^6 e^{-4A_0}}{27\,\ell\mu^6} \int d\Omega_5 \int_{-\infty}^{\infty} dz\, e^{-8|z|/\ell} \\ &\approx \frac{M_{10}^8}{\pi^3} \frac{16\sqrt{2}\,\rho^6 e^{-4A_0}}{729\,\mu^6}. \end{aligned} \tag{62}$$

In the above we made the approximation $\int_{-\infty}^{\infty} dz\, e^{-8|z|/\ell} \approx \ell/4$, which is reasonably good when $z \to \infty$. Note that the warping becomes stronger away from the brane at $z = 0$. This feature is similar to that in RS single brane model.

A. Positive energy condition

For several explicit solutions given above, inflationary cosmology is possible without violating any energy condition in the full D-dimensions. To quantify this, we can make an ansatz for the stress-energy tensor of the form

$$T_A^B = \tau_p P(g_A^B) + \mathcal{T}_A^B. \tag{63}$$

$\mathcal{T}_A^B$ represents the contribution of bulk matter fields. In $D = 5$ dimensions, and with $\gamma = 2$, we have

$$R_5{}^5 - R_0{}^0 = \left[\frac{6W'^2}{W^2} - \frac{3W''}{W} - \rho^2\,\frac{3\ddot{a}}{a}\right] \frac{1}{\rho^2 W^2},$$

where $W(z) = e^{-\mu|z|}$. In order not to violate the 5D null energy condition (NEC) we require $R_5{}^5 - R_0{}^0 \ge 0$. In the simplest case that $\mathcal{T}_{AB} = 0$, we find $R_5{}^5 - R_0{}^0 = 6\mu/\rho^2 > 0$ on the brane and $R_5{}^5 - R_0{}^0 = 0$ in the bulk. Similarly, for the 10D solution given above, Eq. (57), we find

$$\begin{aligned} &\tilde{R}_{mn}\tilde{g}^{mn} - R_0{}^0 \\ &\quad = \left[15 - \frac{57\ell^2}{8}\,\frac{W'^2}{W^2} - \frac{39\ell^2}{8}\,\frac{W''}{W} - \frac{\rho^2}{\mu^2}\,\frac{3\ddot{a}}{a}\right] \frac{\mu^2}{\rho^2 W^2}, \end{aligned}$$

where $W(z) = e^{-|z|/\ell}$. Again, $\tilde{R}_{mn}\tilde{g}^{mn} - R_0{}^0 = 0$ in the bulk and > 0 on the brane. There is no violation of any energy condition in the full D-dimensional spacetime, and no violation of the null energy condition in four dimensions. This result can be understood also from the viewpoint that the NEC can be violated only by introducing non-standard bulk matter fields (i.e. $\mathcal{T}_0{}^0 < 0$) or by introducing negative tension branes or orientifold planes [15] that may serve as sources of the NEC violation in a subspace of the internal manifold.

The above explicit results may appear in conflict with a claim made in [30], where it was argued that to get a four-dimensional de Sitter space solution one may have to violate the five- and higher-dimensional null energy conditions or allow a time-dependent Newton's constant or even both. There is perhaps no contradiction here, since the discussion in [30] may apply only to a particular model with physically compact extra spaces, supplemented with additional constraints on the warp factor. String theory can, of course, accommodate some NEC violating objects, such as negative tension branes and orientifold planes [15], but in our view such objects are not necessary just to get an accelerating universe from higher-dimensional Einstein's theory.

In conclusion, we have proposed an alternative scenario to conventional explanation to cosmic acceleration, by embedding a four-dimensional de Sitter spacetime into higher-dimensional spacetimes. We have shown the existence of inflationary cosmology in a wide class of metrics, obtaining explicit cosmological solutions both in five- and ten-dimensions, which do not violate the higher-dimensional positive energy condition. In $D = 10$ dimensions, our solutions correspond to the dimensional reduction to four dimensions of $d = 10$ supergravity (with zero flux), where the spacetime is a warped product of a four-dimensional de Sitter space dS_4 and a six-dimensional

Einstein space E_6 (with arbitrary curvature). We only took into account contributions from brane sources and metric flux (arising as a nontrivial effect of the internal curvature), so the present construction may be viewed as a local model. The no-go arguments for de Sitter solutions as simple as the one given for classical supergravities with fluxes [11, 12] or the one for string flux compactifications [15] may not be applied to our examples because we considered less symmetric spacetimes with arbitrary curvatures, and also relaxed some of the conditions imposed on the warp factor.

One of the remarkable features of our model is that the brane tension is induced not by a bulk cosmological constant but by the curvature related to the expansion of the physical $3+1$ spacetime, which appears to vanish only in the limit where the scale factor becomes a constant. The universe accelerates when $\mu > 0$, giving rise to a nontrivial warp factor, and the brane tension becomes positive. There is no static limit of our solutions: the scale factor (of the universe) becomes a constant (in a spatially flat FRW universe) only when $\mu = 0$, but in this case the warp factor is also constant and the brane tension vanishes. In the generic situation with a Z_2 symmetry about the brane's position, and with a nonzero Hubble parameter, the four-dimensional Newton's constant is effectively finite.

Within our model the four-dimensional effective cosmological constant is given by $\Lambda_4 = 6\mu^2/\rho^2$, to leading order. It is clearly determined in terms of two length scales: one is a scale associated with the size of extra dimensions or the compactification scale, ρ, and the other is a scale associated with the curvature related to the expansion of the physical three spaces, which also determines the slope of warp factor. This could just be due to a general fact that the warp factor relates energy scales on compactified spaces to those in $3+1$ spacetime.

We conclude with the following remark. Recently, important steps have been taken in the literature toward investigating minimal de Sitter solutions in type IIA and IIB string theory [31, 32, 33]. In most of these works, one adopts a common notion that the low energy effective potential (and hence the gravitational vacuum energy density of our universe) is a sum of the effects from different regions of the internal manifold, supergravity fluxes and effects of localized sources like branes and orientifold planes, and then check certain conditions under which the effective potential allows one or more metastable de Sitter minima. Though this exercise seems reasonable from a viewpoint of effective field theory, it would be more beneficial to know some explicit cosmological solutions at least within some workable models; we leave the analysis of this nature, specific to our metric choices, for subsequent work.

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